International Journal of Theoretical Physics, Vol. 42, No. 9, September 2003 (© 2003)

Laws, Symmetries, and Reality

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Received April 24, 2003

It is argued that quantum mechanics follows naturally from the assumptions that there are no fundamental causal laws but only probabilities for physical processes that are constrained by symmetries, and reality is relational in the sense that an object is real only in relation to another object that it is interacting with. The first assumption makes it natural to include in the action for a gauge theory all terms that are allowed by the symmetries, enabling cancellation of infinities, with only the terms in the standard model observable at the energies at which we presently do our experiments. In this approach, it is also natural to have an infinite number of fundamental interactions.

KEY WORDS: laws; symmetries; reality.

1. INTRODUCTION

We are told that nature is fundamentally "capricious." For example, if we prepare many identical atoms in the *same* excited state at a given time, then they decay at different times, in general. This has been a source of tremendous surprise, if not shock, for many physicists who were brought up to accept, without question, the paradigm of laws (Anandan, 1999). They believe that all physical systems are governed by the same laws, which should compel identical systems having the same initial conditions to behave in the same way. But if we are unbiased by centuries of traditional physics conditioning, then we should actually be surprised if all the atoms behave in the same way. Why should they? After all, they do not communicate with each other to ensure that they would all decay at the same time. And there is no evidence of a "cosmic rail road," i.e., fundamental laws of nature, that would compel all the atoms to behave in the same way (Anandan, 2002).

The view that there are fundamental laws worked very well in classical physics until it was found that the laws of classical physics are really not fundamental. When physicists found that the laws of classical physics were in disagreement with

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observation, they decided to replace these laws with new laws. Although they discarded the existing laws, they did not give up the belief that there must be laws. Newton's second law of motion was replaced by Schrodinger's law that was supposed to govern the evolution of a quantum state. While this law is supposed to govern the *unobserved* evolution of the state between measurements, it does not appear to govern the *observation* of a quantum state, i.e., the process of measurement.

Some physicists found this state of affairs to be highly unsatisfactory. They wanted the quantum laws to apply to every physical process including the process of measurement. Everett (1957), for example, postulated that the Schrödinger law applied to every quantum evolution, which implied that the wave function never collapsed. Bohm (1952), on the other hand, regarded the wave function and the particle to be real, the wave function playing the dual role of giving the probability density for the particle and guiding the particle's motion according to a quantum law. Another approach, pioneered by Pearle (1986, 1989), was to modify Schrödinger's law to a new nonlinear law that would apply to the measurement process.

These and similar approaches to quantum theory were based on two assumptions: (A) there are quantum laws that apply to every physical process, including the measurement process, and (B) a system may exist by itself and its reality does not depend on its interaction with other systems, which I shall call the assumption of absolute reality. But the "capriciousness" of nature mentioned at the beginning suggests that the assumption (A) may not be valid. The great difficulty in applying quantum laws to the observed measurement process suggests that these laws may not apply also to the unobserved state evolution in between measurements. The strangeness of assumption (B) is seen if we imagine a universe consisting of only one object. What is the metaphysical difference between this object existing and not existing? But if there are two objects then these two objects may interact, and each object would then have reality with respect to the other.

It seems reasonable therefore to suppose, instead, that (1) there are no fundamental causal laws but only probabilities for physical processes that are constrained by symmetries, and (2) reality is relational in that an object is real only in relation to another object that it is interacting with. These two assumptions help us to understand why the world is quantum mechanical. Assumption (1) implies that nature must necessarily be indeterministic or "capricious," which is consistent with observed quantum phenomena. Assumption (2) explains why the act of measurement brings into being the state of a quantum system. This is consistent with the Copenhagen interpretation that denies absolute reality. But the present interpretation goes beyond the Copenhagen interpretation by replacing the absolute reality with relational reality. This allows for an objective reality that is relational. According to the present view, a tree falls in a forest even when there is no one to observe it, because of the interactions between the molecules constituting the tree and their interaction with the environment.

A version of relational reality was proposed by Rovelli (1996) and Mermin (1998a,b), who argued for the reality of "correlations without correlata." But this interpretation of quantum mechanics seems to be indistinguishable from the Everett interpretation, which also has all the correlations in the wave function of the universe. The present interpretation differs from these interpretations in two ways: First the relational reality is associated with the *interactions* and not with correlations. For example, the EPR correlation between two noninteracting spin-half particles is due to an interaction that the two particles have undergone in the past. Therefore, the measurements on an ensemble of such pairs that confirm these correlations are due to the earlier interaction that the particles underwent. Second, the implication of assumption (1) that the world is indeterministic introduces probabilities from the very beginning. Whereas obtaining quantum probabilities in the deterministic picture of Everett is a major problem.

In section 2, I shall discuss the fundamental role played by the Poincare group of symmetries in quantum mechanics, and argue that symmetries are more basic than laws. This argument will be extended, in section 3, to include gauge symmetries. I shall then show, in section 4, that this point of view naturally leads to quantum mechanics. Relational reality will be used to justify the Born rule for obtaining quantum probabilities. Finally, in section 5, I shall argue, on the basis of assumption (1) that there should not be a finite symmetry group that gives all the fundamental interactions. This suggests that there must be an infinite number of gauge interactions associated with the groups $SU(N)$, $N = 1, 2, 3 \ldots$.

2. MYSTERIES AND SYMMETRIES IN QUANTUM MECHANICS

One of the most mysterious aspects of quantum mechanics is the *wave-particle duality*. The state of the system may be in an approximate eigenstate of momentum p, in which case it may be regarded as a wave, or in an approximate eigenstate of position x, in which case it may be regarded as a particle. The wave particle duality is therefore due to x and p being independent observables in quantum theory. Closely related to this aspect is *complementarity* that is implied by the Heisenberg commutator the relation:

$$[x_j, p_k] = i\hbar\delta_{jk}, \qquad j, k = 1, 2, 3. \tag{1}$$

This is unlike in Newtonian physics where the momentum is defined by $p = m\frac{dx}{dt}$, and therefore x and p are not independent. Moreover, x and p commute in classical physics.

Both aspects in quantum theory may be understood by realizing that x and p are independent generators of the inhomogeneous Galilei group that is the symmetry group of nonrelativistic quantum mechanics. p generates spatial translation and x generates Galilei boost, and therefore they are independent, which gives rise to the wave-particle duality. During a measurement, what is observed is the *relation* between the apparatus and the observed system. This relation therefore should be regarded as the observable. This is the fundamental reason why observables are operators in quantum theory. These relations are elements of the symmetry group that constitutes the quantum geometry (Anandan, 1999).

To understand the complementarity between x and p, consider the Poincare Lie algebra relations:

$$[K_j, T_k] = i\delta_{ik}T_0, \quad j, k = 1, 2, 3, \quad [T_\mu, T_\nu] = 0, \tag{2}$$

where iK_j generate Lorentz boosts and iT_μ, $\mu = 0, 1, 2, 3$ generate space-time translations. Here K_j are dimensionless because they get multiplied by the dimensionless parameters v/c and exponentiated to obtain the infinitesimal Lorentz boosts, and T_μ have the dimension of 1/length because they get multiplied by distances and exponentiated to get the translations, and the exponents of course must be dimensionless. To relate T_j to the momentum p_k that is conserved under translation, it is therefore necessary to introduce a new scale that has the dimension of momentum $\times$ length. This new fundamental constant, denoted $\hbar$, enables also the time translation T_0 to be related to the energy p_0 that is conserved because of time translational symmetry. We then write

$$\hbar T_j = p_j, \quad j = 1, 2, 3, \quad \hbar c T_0 = p_0. \tag{3}$$

The introduction of Planck's constant here seems to be related to the space-time description in physics.

The Lorentz transformations are generated by $L^{\mu\nu} = x^\mu T^\nu - x^\nu T^\mu$. Then

$$K_j = L^{j0} = x^j T_0 + x^0 T_j. \tag{4}$$

Substituting this in the first relation in (2) and using (3),

$$[x^j, p_k]p_0 = i\hbar\delta_{ij}p_0, \qquad j, k = 1, 2, 3. \tag{5}$$

Now $\eta_{\mu\nu}T^\mu T^\nu = \hbar^{-2}c^{-2}(p_0^2 - \mathbf{p}^2)$ is a Casimir operator that commutes with the Poincare group. For a given irreducible representation, we may therefore set $p_0^2 - \mathbf{p}^2 = m^2c^4$, where m is the mass. This implies that at low energies, $p_0 \approx mc^2$. Then (5) becomes (1). We could have obtained (1) by writing $T_\mu = i\frac{\partial}{\partial x^\mu}$ and using (3). But the purpose of the above exercise is to show that (1), which is so fundamental to quantum mechanics, and the Heisenberg uncertainty principle of that follows from it, ultimately comes from the Poincare Lie algebra relations (2).

Consider now the Poincare Lie algebra relation

$$[K_j, T_0] = iT_j, \qquad j = 1, 2, 3. \tag{6}$$

On using (4) and (3), this reads

$$\frac{1}{c^2}[x^j, p_0]p_0 = i\hbar p_j, \qquad j = 1, 2, 3. \tag{7}$$

Writing $p_0 = mc^2 + H$, at low energies

$$[x^j, H]m = i\hbar p_j, \qquad j = 1, 2, 3. \tag{8}$$

This is satisfied by $H = \frac{1}{2m}\mathbf{P}^2 + V$, where V commutes with x^j. It follows also from the Poincare Lie algebra relations that $[p_j, H] = 0$ and $[J_k, H] = 0$, where J_k generate rotations. However, the above assumption of Poincare symmetries implies that this nonrelativistic Hamiltonian H corresponds to an isolated system that is noninteracting.

The above procedure leads to the Inönü–Wigner contraction (Inönü and Wigner, 1952) of the Poincare group to the quantum mechanical inhomogeneous Galilei group. It was shown by Jauch (1968) that the covariance of time evolution under the homogeneous Galilei group requires that the Hamiltonian that generates time evolution must be of the form

$$H = \frac{1}{2m}[\mathbf{p} - \mathbf{A}(\mathbf{x}, t)]^2 + V(\mathbf{x}, t). \tag{9}$$

Thus symmetries determine the ‘law’ of quantum evolution as well as the interaction of the quantum state that includes the electromagnetic interaction.

Another important consequence of the Poincare symmetries is seen by taking the expectation value of with respect to the vacuum of the first of the relations (2) with $j = k$:

$$\langle 0|[K_j, T_j]|0\rangle = i\langle 0|T_0|0\rangle. \tag{10}$$

Since the vacuum is invariant under translations or boosts, the left-hand side is zero. Therefore, $\langle 0|T_0|0\rangle = 0$, which implies that the vacuum energy is zero. But, as is well known, if we apply the laws of quantum mechanics to fields then the vacuum energy is *infinite* due to the fact that the fields consist of infinite number of harmonic oscillators that have zero point of energies. This shows that the symmetries should be regarded as more fundamental than the “laws” that are applied to these harmonic oscillators.

It is now tempting to say that the above fundamental role played by the Poincare and Galilei groups in giving rise to quantum mechanics is due to the existence of space-time on which these groups act as symmetries. But the experimentally observed intrinsic spin suggests that the symmetries are more fundamental then the space-time description. The generators J_k of the rotation group $SO(3)$ that is a subgroup of the Poincare group are the components of angular momentum. The commutator relations of these components are therefore the same as the Lie algebra relations of the rotation group. The intrinsic spin that is contained in these generators cannot be obtained from the space-time description that gives only the orbital angular momentum. Also, as is well known, in this way only the integer spin particles, or Bosons, are obtained. To obtain half integer spin particles, or Fermions, it is necessary to postulate $SU(2)$ that is the covering group of $SO(3)$

as the symmetry. We cannot regard a Fermionic wave function as "immersed" in space-time because when it is rotated by 2π radians it changes sign, which has observable consequences (Aharonov and Susskind, 1967). The $SU(2)$ group, which contains this transformation, cannot therefore be associated with the symmetries of space. Also, the addition rules for angular momenta of two systems can only be understood by considering irreducible representations of $SU(2)$ group in the tensor product of the Hillbert spaces of the two systems, and not by regarding angular momentum vector as representing rotation of matter in space about an axis with arbitrary direction. To obtain this group, the usual space-time Poincare group P needs to be replaced by the semidirect product $\tilde{P}$ of $SL(2, C)$ and the translational group. And $SU(2)$ is a subgroup of $SL(2, C)$. Since $\tilde{P}$ is not a direct consequence of the usual space-time description, we should regard $\tilde{P}$ as being more fundamental than space-time.

3. STRUCTURE OF A GAUGE THEORY

More generally, in relativistic quantum theory, the quantum system interacts with a general gauge field. The gauge symmetry group implies, via Noether's theorem, conserved quantities. These conserved quantities generate fields, which is believed to be in accordance with a "law." For example, in electromagnetism the gauge symmetry group is $U(1)$ and the conserved quantity is the electric charge that generates the electromagnetic field in accordance with Maxwell's law. The great achievement of Weyl and Yang-Mills was to recognize that the third side of this triangle (Fig. 1) may be completed by means of the gauge principle, i.e., the gauge fields may be obtained directly from the gauge group by requiring local gauge symmetry.

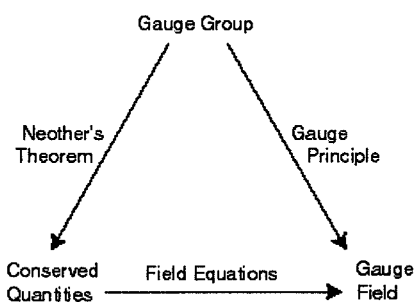

Fig. 1. The structure of a gauge theory. The fundamental role played by the gauge symmetry group is shown from the fact that the gauge field may be obtained from the gauge group via the conserved quantities or directly by means of the gauage principle. The requirement that both paths are equivalent suggests that all possible invariants may play a role in the Lagrangian for the gauge field.

The first side of this triangle, namely Noether's theorem that gives the conserved quantities from the symmetries, is tautological (Anandan, 2002) as mathematical theorems are. The third side may be made nearly tautological if the gauge principle is implemented not in a particular Lagrangian but as follows: To compare vectors that belong to internal spaces at different space-time points, it is necessary to introduce a connection that enables parallel transport of a vector from one point to another. In fact, the gauge field may be introduced by giving the holonomy transformations as elements of the symmetry group from which the connection may be reconstructed, and the connection is then unique up to gauge transformations (Anandan, 1983, 1986a). This shows that gauge fields may be obtained directly from symmetries without a dynamical or causal law. On the other hand, for the second side, it is usually supposed that the conserved quantity rigidly determines the gauge field according to a "law." This would make the second side fundamentally different from the first and the third sides.

But I shall suppose now that going from the first vertex (symmetries) to the third vertex (gauge field) along the third side is in some sense equivalent to going along the first and the second sides. This implies that the second side should not pick out a particular form of the Lagrangian or Hamiltonian like the first and the third sides. Therefore, we should *not* suppose that the Lagrangian for the gauge field is the Yang-Mills Lagrangian that is proportional to the Lorentzgauge invariant

$$I_1 = F^i_{\mu\nu} F^{i^{\mu\nu}}. \tag{11}$$

For the $SU(2)$ gauge field, for example, any Lorentz-gauge invariant that is a polynomial in the field strength $F^i_{\mu\nu}$ is a polynomial function of 10 Lorentzgauge invariants that are polynomials in $F^i_{\mu\nu}$, of which only 9 are independent. The simplest of these invariants is (11). But there are other invariants such as $I_2 = \epsilon_{ijk} F^i_{\mu\nu} F^{j^{\nu\rho}} F^{k\mu}_{\rho}$ or $I_3 = F^i_{\mu\nu} F^{j^{\mu\nu}} F^j_{\rho\sigma} F^{i^{\rho\sigma}}$ on which the Lagrangian may depend on. I shall therefore suppose, in accordance with the above hypothesis, that the Lagrangian depends on all 10 invariants. More generally, for an arbitrary gauge theory, I shall assume that the Lagrangian is a function of all the Lorentz-gauge invariants.

This makes all three sides of the triangle in Fig. 1 similar in the sense that none of them pick out a particular invariant to determine the dynamics. However, I do not have an explanation for the values of coupling constants that should appear in the Lagrangian apart from appealing to a version of the anthropic principle: There are parallel universes that constitute what I have called polyverse (Anandan, 2002), and it then follows that we must live in a universe with coupling constants that enable life to evolve.

According to the standard model, three of the four fundamental interactions that are known today are described by a gauge theory with the gauge group $U(1) \times SU(2) \times SU(3)$. What about the remaining interaction, namely gravitation?

From an experimental point of view, the best “law” that we have for gravitation are the classical Einstein’s field equations. But this “law” does not determine the signature of the metric. The Lorentzian signature of the metric, however, is obtained by requiring invariance of the metric at each point under the Lorentz group of symmetries. This suggests that for gravity as well symmetries may be more fundamental than laws. Indeed, it is possible to characterize the gravitational field by associating an element of the Poincare group with each (piecewise differentiable) curve on space-time (Anandan, 1996) analogous to characterizing a gauge field by associating gauge group elements with each such curve (Yang, 1974).

However, hitherto it has been held that a fundamental difference between gravity and gauge fields is due to the Lagrangian for gauge fields being quadratic in the curvature or field strength whereas the Einstein–Hilbert Lagrangian for gravity is linear in the curvature. But the above hypothesis that the two paths to gauge field from the gauge group should be equivalent removes this fundamental distinction. This is because, according to this hypothesis, all the invariants should be included in the Lagrangian for both gravity and gauge fields.

It is well known that the Einstein–Hilbert Lagrangian density for gravity $\sqrt{-g}R$, where R is the Rcci scalar, is not renormalizable. If one requires that there should be a “law” given by a Lagrangian that depends on a small finite number of invariants then it becomes necessary to deal with the infinities that arise in Feynman amplitudes by the process of renormalization. But if the Lagrangian depends on all possible invariants then there are counter terms to cancel all the infinities (Weinberg, 1995). The usual process of renormalization that absorbs the infinities in a finite number of coupling constants was conceived within the paradigm of laws because of the belief that the Lagrangian that constitutes the “law” should depend on a small subset of all possible invariants. But if we allow all possible invariants then the infinities may all be cancelled.

The present approach then appears to solve the riddle that arises within the paradigm of laws of why nature should choose particular Lagrangians and not others for the laws. This is because, according to the present view, all lagrangians consistent with a given set of symmetries are allowed. However, in the present approach, symmetries replace the fundamental role previously played by laws. And the “laws” are obtained from the symmetries, as effective laws, instead of the other way around.

4. WHY THE WORLD IS QUANTUM MECHANICAL

It was argued in section 2 that a great deal of quantum mechanics may be obtained from the Poincare group of symmetries. In section 3, this argument was extended to include the gauge symmetries that are used today to describe three of the four known fundamental interactions. From an operational point of view, observing the gauge field by a quantum mechanical probe is the same as observing the holonomy transformations that are elements of the gauge group (Anandan, 1996). Hence, symmetries are directly observable in quantum mechanics. But another important ingredient of quantum mechanics is the linearity of the time evolution of a quantum state, which leads to the quantum measurement problem. Why should the Hamiltonian obtained using symmetries in section 2, for example, generate linear time evolutions of state vectors? It will now be argued that assumption (1), stated in the Introduction, naturally leads to this linear time evolution, or Schrödinger's equation.

Since, according to assumption (1), there are no fundamental causal laws, all the infinite possible ways in which a system may go from an initial state to a final state should have equal probabilities. For this to make mathematical sense *in the absence of laws,* there should be cancellation between the different paths in spite of them having equal probabilities (Anandan, 2002). This becomes possible only by introducing the probability amplitude associated with each path so that to determine the probability of a process the amplitudes should be added for the different ways in which the process can take place and the probability is determined from this sum. Mathematical considerations then suggest that the probability amplitudes should be complex numbers (Anandan, 2002).

The requirement that these probability amplitudes should be invariant under the symmetries, in accordance with assumption (1), then gives quantum mechanics in the Feynman path integral formulation, except that the action that is the phase of the probability amplitude needs to include all possible terms that are invariant under the symmetries. Since the Feynman path integral formulation is equivalent to the Schrödinger formulation, we obtain the linear time evolution of the state vector. Also, including all possible invariants in the action of a gauge theory provides counter terms to cancel all the infinities that arise in summing the amplitudes in the quantum field theory, as mentioned in section 3. But to have consistency with experiments, only the lowest order terms in the action that are in the standard model should be observable at the energies at which we do experiments at present. It can be argued that such a theory would not have unitary time evolution at large energies. But this argument is based on our present understanding of quantum field theory, which may have to be modified at higher energies.

How and *when* do we convert probability amplitudes into probabilities? Quantum mechanics provides a clear answer to the question of "how," namely the Born rule, but is infamously ambiguous about the question of "when." We are told that probability amplitudes should be added or multiplied when no "observation" is made, and that the probability amplitudes should be converted to probabilities when an "observation" is made. But no clear criteria for what constitutes an observation is given, apart from some vague ideas about interaction with a macroscopic system. It is my contention that the answer to the above question of "when" provides also the answer to the question of "how," i.e., a derivation of the Born rule.

In classical physics, the events, for which we can only predict probabilities in quantum physics, have no ontological ambiguity. These events are assumed to exist independently of any observation of them. But even in classical physics, events are due to interaction between two or more objects, such as the Einsteinian example of lightning striking a railway track. The belief in the existence of an object independently of its interaction, which I call absolute reality, cannot be operationally confirmed. Absolute reality is therefore a metaphysical assumption, which cannot even be philosophically defined because there does not exist a philosophical criterion for distinguishing between "existence" and "nonexistence" other than observation. The assumption of absolute reality may be justified if its consequences are confirmed by observation. But the observer dependence of the quantum state suggests that this assumption is not valid.

I shall therefore assume, instead, that reality is relational in the sense that two objects exist in relation to each other if and only if they interact. How can we speak of objects whose very existence is conditional upon their interaction? The statement that "there are no ghosts" does not presuppose the existence of ghosts. So, there is no contradiction in referring to objects that do not exist, although in the present case they would exist in a relational sense when they interact, which necessitates referring to them. Even in classical physics, the reality of the electric field is determined by what it does to a charge; the field is therefore real with respect to the charge that it interacts with. The difference between classical and quantum physics is that different charges respond to a classical field like as if it is the same field, which gives the illusion that the field is independent of its interaction with the charge. Whereas, in quantum physics, the states of two interacting systems become entangled, in general, which should prevent us from assigning independent reality to either state.

Relational reality leads to the experimentally observed Born rule for obtaining probabilities from probability amplitudes, which may therefore be regarded as evidence of relational reality. This is most easily seen in the double slit interference experiment. If the particle going through one of the slits interacts with another physical system, then it is this interaction that brings into reality the particle going through that slit relative to the physical system that it interacts with. If this interaction does not take place then the only way the particle could go through the screen with the double slit is by passing through the other slit. Therefore, for this arrangement, the probability of the particle interacting with any part of the subsequent screen is the sum of the two probabilities for passing through each of the two slits. This requirement naturally leads to the Born rule that this probability is $|\psi|^2$ where ψ is the sum of the two probability amplitudes for the particle to go through the two slits (Anandan, 2002).

A special case of an interaction is when a human being, or more generally an animal being, makes an observation. what is observed then acquires relational reality with respect to this being. We may therefore understand now Wigner's hypothesis that the "consciousness" collapses the wave function (Wigner, 1961) as a special case of relational reality. However, it would be a mistake to assume, as Wigner may have done implicitly, that the collapsed wave function has absolute reality. For example, the Schrödinger cat inside a box may be alive in the sense that different parts of its body are accordingly interacting with each other and with the box. But an observer outside the box may *expect* the cat to be in a superposition of alive and dead states, on the basis of prior measurements, although there is no interaction to verify this expectation due to the large number of degrees of freedom of the cat. There is no contradiction between the two views because reality is relational. However, the cat acquires relational reality only as alive or dead because of the restrictions on the interactions that it could have with another object. But if the cat is replaced by a microscopic system, then the outside observer may observe this system in the expected superposition by means of a suitable interaction, which is now possible because of its small number of degrees of freedom. This provides a resolution of the Schrödinger cat paradox (Anandan, 2002).

Descartes' famous statement "I think, therefore I am" (*Cogito ergo sum*) created, despite Descartes' healthy skepticism of reality, a great deal of confusion in Western philosophy. "I think" means interactions between different parts of the brain, which therefore have relational reality with respect to each other. But concluding from this "I am," implying absolute reality of the self, is an unjustified extrapolation.

5. BEYOND SYMMETRIES

To do physics, we must communicate information. This naturally leads to gauge fields and gravitation (Anandan, 1986b). The symmetry group of the experimentally very successful standard model is the direct product of the Poincare group and the gauge group. This suggests that the gauge group may also just be a direct product of groups, as it already is in the standard model. But if there is one finite dimensional symmetry group that determines all interactions then this would constitute a law. It may well be that the way we observe the universe with the very limited apparati that we have which makes symmetries so useful. At the low energies in which we do our experiments, the symmetry groups $U(1)$, $SU(2)$, $SU(3)$, which are the simplest unitary groups, useful. But at higher energies, we may find the gauge groups. $SU(4)$, $SU(5)$, $SU(6)$, . . . useful.

The gauge group $SU(4)$ was used by Pati and Salam (1973a,b, 1974, 2002) to unify quarks and leptons by putting the three color states of a quark and the corresponding lepton in the same multiplet on which the fundamental representation of

$SU(4)$ acts. Also, the smallest simple group that contains the gauge group of the standard model is $SU(5)$, which has therefore been used in an attempt to unify the electroweak and strong interactions (Georgi and Glashow, 1974). A major problem in such grand unification models is that proton decay has not been observed yet. Also, all the grand unification attempts implicitly assume that there exists a finite dimensional symmetry group that unifies the fundamental interactions. But such a symmetry group would constitute a law, and is therefore contrary to the spirit of the present paper according to which there is no intelligent design to the universe. This suggests that there should be an infinite hierarchy of fundamental interactions associated with $SU(n)$, $n = 1, 2, 3, 4, \ldots$.

The universe on a large scale is held together by the gravitational interaction, which is associated with the Poincare group (Anandan, 1983, 1986a). If we probe deeper on the scale of molecules and atoms, we find that they are held together by electromagnetic interactions, corresponding to the $U(1)$ EM symmetry group. Owing to the success of the standard model, we should say that the electrons and the nucleus in an atom are held together by the electroweak field corresponding to $U(1) \times SU(2)$, which leads to parity violation in atoms. If we probe deeper, we observe the strong interactions that hold the quarks together in neutrons, protons, and other hadrons, associated with the group $SU(3)$. The fact that these are the simplest unitary groups suggests that this may be due to the low energies of the experiments that we have been doing so far. Extrapolating, it would appear that quarks and leptons have constituents that are held together by a gauge field of the $SU(4)$ group. But since quarks and leptons have spin 1/2, we then expect this symmetry group to be broken so that three of the four particles the fundamental representation form the quarks and leptons. This is analogous to how in the Pati–Salam model the $SU(4)$ symmetry is broken so that the hadrons are formed by the quarks, while the leptons stand apart.

Present experiments have placed an upper limit for the radii of quarks and leptons of about 10^{-17} cm. The next generation of experiments in the Lepton–Hadron Collider is expected to probe scales less than 10^{-18} cm. So, there is hope that the above extrapolation to a superstrong force of the $SU(4)$ gauge field may be experimentally testable.

To conclude, we recall Einstein's famous statement that our theories are to the external world what clothes are to the human body. The physical theories proposed so far have all been based on the assumption that there are fundamental laws. This would mean that these laws or "clothes" may be made to fit the objective reality more and more closely, but there is an *unbridgeable gap* between them. The purpose of this paper was to argue that these "laws" are like "the emperor's new clothes." For the relational reality obtained through our observations, laws and, at a deeper level, symmetries are useful. But neither may be a reflection of any fundamental structural realism.

ACKNOWLEDGMENT

I thank Yakir Aharonov, Pawel Mazur, and Parameswaran Nair for useful discussions. This research was partially supported by an NSF grant and an ONR grant.

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